

NONLINEAR MODELING OF VALVES IN A DIFFERENTIAL ALGEBRAIC HYDRAULIC SYSTEM OF TWO-FLOW MIXING

NONLINEAR MODELING OF VALVES IN A DIFFERENTIAL ALGEBRAIC HYDRAULIC SYSTEM OF MIXING OF TWO FLOWS

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Abstract -- This paper shows a differential algebraic model derived from the nonlinear consideration used in the flow of liquid through a valve, defined as the product of a coefficient associated with the geometry of the valve multiplied by the square root of the potential difference. This consideration is used in the two-liquid confluence model with three valves with such nonlinear characteristics. In the algebraic nonlinear part a dynamic algorithm of algebraic solution is proposed and in the dynamic part a pair of tanks in autonomous dynamics is considered without loss of generality. To show the dynamic representation, a numerical simulation is shown where the valves are configured in a time-varying way, with which the nonlinear behaviors of the liquid levels and the outflow are performed.

Keywords: flow mixing, nonlinear valve model, differential algebraic hydraulic system.

Abstract -- In this paper a differential algebraic model with the nonlinear consideration, which is based on the a coefficient weighting of the square root of differential pressure through a valve, such coefficient weighting is associated with the geometry of the valve. This nonlinear valve characteristic is introduced in a dual-flow confluence model with three valves. In the algebraic nonlinear part, a dynamic algebraic solver algorithm is proposed, and in the dynamic part of the model, without loss of generality, a pair of tanks in autonomous dynamics is considered. To show the dynamic representation, a numerical simulation is shown where the valves are configured in a time-varying way, with which the nonlinear behaviors of the liquid levels and the outflow are performed.

Key words: flow mixture, nonlinear valve model, algebraic differential hydraulic system.

INTRODUCTION

On the subject of water saving, we have works in which [17] point out the importance of optimizing resources in view of the increase in the human population in the world, mainly water. In [13], an economizer is presented that has the function of storing water until, by recording its temperature, the user decides to start the shower, thus avoiding water waste; in [10], a thermomechanical device is presented that water waste in appliances and equipment that handle hot water for domestic use, such as showers, bath tubs, washing machines, etc. The authors conclude that it allows an efficient use of water; in [1] a temperature control system for the electric shower was designed, simulated and implemented, managing to maintain the temperature at 35°C and a constant flow rate of 2.2 l/min. Using a control card for the manipulation of an access valve; in [22] an algorithm was developed to analyze the two-phase flow in collection. This is based on the on-line resolution of the hydraulic variables, being a modification of the one initially proposed by [23], who considered the liquid phase as an oil-only unit, and instead contemplates the two-phase oil-water problem. The numerical model allows the analysis and design of pipe networks with multiphase flow, through the implementation of a simultaneous resolution algorithm that does not require the algebraic handling of matrices. As for the proposals of taps in [12], modifications were made to tap threads to optimize tap accuracy; and in [24], a hygienic and energy-saving tap was developed.

for use in public drinking fountains in developing countries.

Several works have been carried out to combat water wastage as in [11] where the measurements, studies, analysis and characterization of equipment necessary to have a complete methodology for the multi-criteria selection of the most appropriate hydro-efficiency systems to equip a given building were presented; in [25], who designed the fuzzy logic controller type 2 to control the outlet temperature as well as the outlet flow of the shower system; in [20] the excessive water consumption in the shower mixers is raised and in turn a possible solution for this is proposed; and finally in [3] the technology of suction irrigation with the use of porous capsules is presented as a viable alternative for its use in garden irrigation with optimal results.

Regarding the topic confluence control in the industry, in [7] the simulation of the automatic control of a paint filling, mixing and packaging system using the PLC as a control element is addressed. Achieving the optimization of the times in which the product can be packaged and launched to the market; in [4] they performed the computational model of the flow dynamics in the Convergent- Divergent Nozzle considering the thermal imbalance and present guidelines for the simulation of fast boiling flow in industrial applications; and in [15] presented a closed-loop volume flow control algorithm for high switching pneumatic valves with PWM signal, the algorithm was tested with the control of the contraction/position of a Pneumatic Artificial Muscle with two fast-switching valves and the minimum requirements of the microcontroller or the

PLC and conclude that bilinear interpolation has the best correlation.

And we can finish with this background with the topic that refers to the use and mathematical representation of the valves, where a nonlinear representation is usually incorporated. The first one is to assume a steady state in which the valves are considered linear in the operating region as in [18] a linear approximation is used to model the valve, or in [16] where for

modeling the transient flow in a pipeline using the frequency response method (FRM) linearizes the significant nonlinearities associated with the friction and valve equations; the second is to address the nonlinear behavior of valves as in [19] which proposes a nonlinear model for a pneumatic process control valve, the controller proposed by [8] is able to solve the problems related to the uncertain and nonlinear behavior of the pneumatic control valve in the flow control loop. On the other hand, in [21] a Managed Pressure Drilling, (MPD) control system for the choke valve is proposed that uses a nonlinear model predictive controller and a two-phase flow model to calculate the annular pressure and dynamics of the choke valve during the water hammer control operation. The performance of the proposed model is demonstrated by simulating the control operation comparing its results with a single-phase linear model. To this problem can be added the challenges associated with the various types of valves such as microvalves which are studied in [2], conical valve, disc valve and compensated valve which are studied in [5], and in [9] valve control through friction compensation are addressed.

Therefore, an area of opportunity can be seen that can be useful in to the adequate use of water and can also be associated to an adequate consumption of energy, in the sense that an adequate use of water also avoids waste of resources. In this sense, the objective of this article is to develop a two-flow mixing model of two exogenous pressure sources, considering the nonlinear nature of the effect of hydraulic valves with respect the difference in pressures at the inlet and outlet of the valve.

The contribution of this paper lies in the proposal of a differential algebraic model, where the valves involved are not shown linearized at an operating point, which allows the dynamic representation of the mixing of two flows in a way that may be more attractive to those who need dynamic representations in larger regions of operation. This rationale, in turn, generates interesting

motivations for alternative or derivative control challenges, where, to mention one example, mixtures of flows at different temperatures or densities may be addressed.

DEVELOPMENT

This section contains the development divided into five subsections which are, the model of the flow of a valve, the development of the algebraic model, taking advantage of the properties of this model, a proposed numerical solution of the algebraic model is defined in a section dedicated to the numerical solution algorithm, in the penultimate subsection the differential algebraic model proposed in this article is proposed; and at the end of the section a numerical simulation is shown in which changes in the opening of the valves are configured.

Valve flow model

The flow rate of a valve in its most compact model can be found as in equation (1) adapted from [6] is:

$$q = K_v \sqrt{\frac{P_a - P_b}{G}} \quad \text{Eq. (1)}$$

Where

- q - Flow through the valve.
- K_v - Valve flow coefficient y its units are $m^3/(s \cdot Pa^{0.5})$ for when the liquid is water.
- G - Relative density of the liquid with respect to that of water.
- P_a - Valve inlet pressure
- P_b - Pressure at the outlet of the same valve.

Remark 1.- Note that, if the fluid through the valve is water, a simpler model is obtained, and if it is considered that the valve will be manipulated, a model like the one shown in Eq (2) can be proposed.

$$q = u \sqrt{P_a - P_b} \quad \text{Eq. (2)}$$

where u is a variable that is manipulated from zero aperture, up to full aperture, or in other words

words, you can obtain values defined in the following interval:

$$0 \leq u \leq K_v. \quad \text{Eq. (3)}$$

Algebraic model of pipes and valves

To address the nonlinear algebraic model of the valves, consider the schematic in Figure 1.

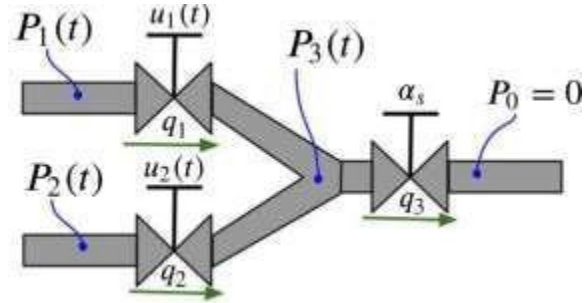


Figure 1. Schematic diagram of the valves

In this scheme, the following variables and parameters are found $P_1(t)$ and $P_2(t)$ are the pressures at the pipe inlet, and q_1 and q_2 are the flows at through the respective valve, $P_3(t)$ is the pressure of the liquid at the inlet of valve 3, through which the flow passes q_3 . The valves are configured through the parameters u_1 and u_2 and which depend on the geometry and construction of the valve.

The flow of the three valves is described by equations (4), (5) and (6)

$$q_1 = u_1 \sqrt{P_1 - P_3}; \quad \text{Eq. (4)}$$

$$q_2 = u_2 \sqrt{P_2 - P_3}; \quad \text{Eq. (5)}$$

$$q_3 = \alpha_3 \sqrt{P_3}; \quad \text{Eq. (6)}$$

By the conservative property of the mass [14], the outflow is expressed by equation (7)

$$q_3 = q_1 + q_2. \quad \text{Eq. (7)}$$

It is considered that the flows do not change direction, i.e.

$$P_3 < \min\{P_1, P_2\} \quad \text{Eq. (8)}$$

and that pressures are always positive

$$P_1, P_2 > 0; \quad \text{Eq. (9)}$$

Equations (4), (5) and (6) are substituted into (7):

$$\alpha_3 \sqrt{P_3} = u_1 \sqrt{P_1 - P_3} + u_2 \sqrt{P_2 - P_3} \quad \text{Eq. (10)}$$

Equation (10) is squared and rearranged:

$$\begin{aligned} (u_1^2 + u_2^2 + \alpha_3^2) P_3 - (u_1^2 P_1 + u_2^2 P_2) &= \\ = 2u_1 u_2 \sqrt{P_1 - P_3} \sqrt{P_2 - P_3} \end{aligned} \quad \text{Eq. (11)}$$

The left-hand side of equation (11) is a straight line with respect to P_3 and is renamed as follows

$$r(P_3) := (u_1^2 + u_2^2 + \alpha_3^2) P_3 - (u_1^2 P_1 + u_2^2 P_2) \quad \text{Eq. (12)}$$

From this equation the following is determined:

$$\frac{\partial r(P_3)}{\partial P_3} = u_1^2 + u_2^2 + \alpha_3^2 > 0 \quad \text{Eq. (13)}$$

$$r(0) = -(u_1^2 P_1 + u_2^2 P_2) \quad \text{Eq. (14)}$$

The right-hand side of equation (11) is represented as follows:

$$c(P_3) := 2u_1 u_2 \sqrt{P_1 - P_3} \sqrt{P_2 - P_3} \quad \text{Eq. (15)}$$

Remark 2.- Note that equation (15) in the pipe operation interval, i.e. when

$$\forall P_3 : 0 < P_3 < \min \{ P_1, P_2 \} \quad \text{Eq. (16)}$$

it is verified that $c(P_3)$ is decreasing

$$\frac{\partial c(P_3)}{\partial P_3} = 2u_1 u_2 \left(\frac{\sqrt{P_1 - P_3}}{\sqrt{P_2 - P_3}} + \frac{\sqrt{P_2 - P_3}}{\sqrt{P_1 - P_3}} \right) < 0 \quad \text{Eq. (17)}$$

and with a range defined by

$$0 < c(P_3) < 2u_1 u_2 \sqrt{P_1 P_2} \quad \text{Eq. (18)}$$

Figure 2 shows the functions $c(P_3)$ and $r(P_3)$ showing the characteristics previously

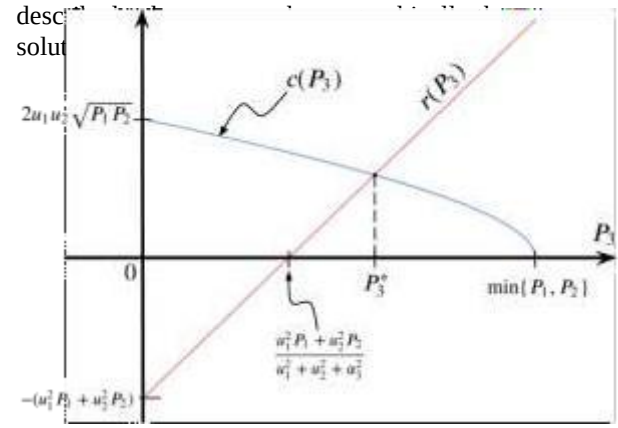


Figure 2. Graph of the solution for P_3 in the operating range

In turn, figure 2 shows that the only solution to equation (16)

$$r(P_3) = c(P_3), \quad \text{Eq. (19)}$$

is when

$$P_3 = P_3^* \quad \text{Eq. (20)}$$

Observation 3.- From the domain of operation and the corresponding ranges of $c(P_3)$ and $r(P_3)$, we can conclude two things associated to the existence of the solution for P_3 , which are:

- a) The solution for P_3 is within the interval expressed in (18),

$$\frac{u_1^2 P_1 + u_2^2 P_2}{u_1^2 + u_2^2 + \alpha_3^2} < P_3 < \min \{ P_1, P_2 \} \quad \text{Eq. (21)}$$

- b) For the solution to exist, the valves together with the pressures must satisfy the following inequality

$$\frac{u_1^2 P_1 + u_2^2 P_2}{u_1^2 + u_2^2 + \alpha_3^2} < \min \{ P_1, P_2 \} \quad \text{Eq. (22)}$$

Numerical solution algorithm for pressure

P_3

The algorithm in difference equations expressed in the system in (23), is a

proposed representation for the midpoint or bisection method of numerical solution.

$$\Sigma_{SN} : \begin{cases} \left[\frac{P_{\bar{3}_{k+1}}}{P_{\bar{3}_{k+1}}} \right] = \left[\frac{P_{\bar{3}_k} + \bar{P}_{\bar{3}_k}}{2} \right] - \left[\frac{\frac{P_{\bar{3}_k}}{2}}{\frac{P_{\bar{3}_k} - \bar{P}_{\bar{3}_k}}{2}} \right] \cdot \sigma \left(h \left(\frac{P_{\bar{3}_k} + \bar{P}_{\bar{3}_k}}{2} \right) \right) \\ \bar{P}_{\bar{3}_k} = \frac{P_{\bar{3}_k} + \bar{P}_{\bar{3}_k}}{2} \end{cases} \quad \text{Eq. (23)}$$

where

$$\sigma(\rho) = \begin{cases} 1, & \rho \geq 0 \\ 0, & \rho < 0 \end{cases}, \quad \text{Eq. (24)}$$

is the binary function that defines whether ρ is greater than or equal to zero, with a 1; and with a 0 if $h(\rho_3)$ is negative.

where

$\underline{P}_{3_k}$ - Lower value of the approximation for P_3 at iteration k .

$\bar{P}_{3k}$ - Upper value of the approximation for P_3 at iteration k .

$\bar{P}_{3k}$ - Mean value between the upper and lower value of the approximation for P_3 in the iteration

k.

And the equality to be solved in (11) is redefined in algorithm (23) as follows

$$h(P_3) = c(P_3) - r(P_3); \quad \text{Eq. (25)}$$

i.e.

$$h(P_3^*) \cong 0. \quad \text{Eq. (26)}$$

If algorithm (23) is used, and the initial conditions of the algorithm are selected as equations (27) and (28):

$$\underline{P}_{3_0} = \frac{u_1^2 P_1 + u_2^2 P_2}{u_1^2 + u_2^2 + \alpha_3^2} \quad \text{Eq. (27)}$$

$$\bar{P}_{3_0} = \min \{P_1, P_2\} \quad \text{Eq. (28)}$$

then $\bar{P}_{j_2}$ converges to the solution, ie,

$$P_3^* = \lim_{k \rightarrow \infty} \widetilde{P}_{3k}, \quad \text{Eq. (29)}$$

Main result: A differential algebraic hydraulic model.

The dynamic model that is coupled to the pipeline described above consists of a pair of tanks, of cross-sectional area equal to A , so that

the pressure P_1 is provided by the first tank, and the pressure P_2 is provided by the second tank, expressed as

$$P_i = \beta x_i, \quad i = 1, 2; \quad (\text{Eq. 30})$$

These pressures depend on the liquid level x_1 for the first tank and x_2 for the second tank, and the parameter is defined as $\beta = 9,806.38$, for when the liquid is water.

The differential algebraic hydraulic system proposed in this paper is shown below.

$$\Sigma_H : \begin{cases} \dot{x}_1 = -\frac{u_1}{A} \sqrt{\beta x_1 - P_3} \\ \dot{x}_2 = -\frac{u_2}{A} \sqrt{\beta x_2 - P_3} \\ 0 = -a_s \sqrt{P_3 + u_1 \sqrt{\beta x_1 - P_3} + u_2 \sqrt{\beta x_2 - P_3}} \\ y = u_1 \sqrt{\beta x_1 - P_3} + u_2 \sqrt{\beta x_2 - P_3} \end{cases}$$

Eq. (31)

Figure 3 shows the schematic diagram representing this system.

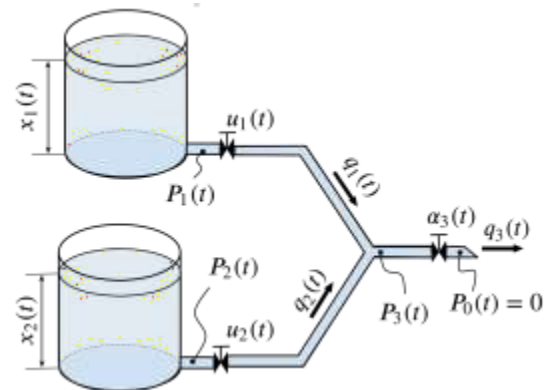


Diagram of the differential algebraic hydraulic system of Eq. (31).

The system has two dynamic equations that represent the behavior of the liquid level of each tank, an algebraic equation, whose solution defines the pressure. The manipulated input variables are the valve opening setting of valve 1 through u_1 , and the valve opening setting of valve 2 through

The output of the system is the flow through valve 3, and is represented by the variable y .

Observation 4.- The differential algebraic hydraulic system shown in the system (31), if it had been proposed considering linear models in the valves, then dynamically it would be a linear system, and moreover, it would be expressed as an explicit function for the P_3 valves. y would be expressed as an explicit function for y , before nonlinear model of the valves considered in this article, in the third expression of the system (31) is necessarily defined as an implicit function, and a numerical method must be used to numerically define the solution for P_3 . For this reason, use is made of the algorithm expressed in (23).

An algorithm showing how a numerical simulation of the system in (31) can be carried out is shown below.

Step 1: Configure the simulation time T where $0 \leq t_j \leq T$, the integration step δ , the parameters β , α_3 and the error tolerance $\epsilon > 0$.

Step 2: $j = 1$;

Step 3: If $j \cdot \delta \geq T$, then go to step 18, otherwise go to step 4.

Step 4: $t_j = (j - 1) \cdot \delta$;

Step 5: Evaluate the functions at t_j , for the valve $u_1(t_j)$ and at $u_2(t_j)$

Step 6: $k = 1$;

Step 7: $P_1(t_k) := \beta \cdot x_1(t_k)$; $P_2(t_k) := \beta \cdot x_2(t_k)$;

Step 8: If $\frac{u_1^2(t_k)P_1(t_k) + u_2^2(t_k)P_2(t_k)}{u_1^2(t_k) + u_2^2(t_k) + \alpha_3^2} > \min\{P_1(t_k), P_2(t_k)\}$ then go to step 17 (no solution for or some pressure is zero); otherwise, go to step 9.

Step 9: If $|c(P_3) - r(P_3)| > \epsilon$ then go to step 14; if not, go to step 10.

Step 10: Calculate $\begin{bmatrix} \bar{P}_{3k+1} \\ \bar{P}_{3k+1} \end{bmatrix}$ as shown in Eq. (23).

Step 11:

Step 12: $\bar{P}_{3k} = \frac{\bar{P}_{3k} + \bar{P}_{3k}}{2}$

Step 13: Return to step 9.

Step 14: $P_3(t_j) := \bar{P}_{3k}$.

Step 15: Calculate $\begin{bmatrix} x_{1k+1} \\ x_{2k+1} \end{bmatrix}$ with the numerical integration method chosen for the differential part of the system (31).

Step 16: $j := j + 1$.

Step 17: Go to step 3.

Step 18: End.

The following subsection is devoted to show the simulation results of the system.

Numerical simulation of hydraulic, differential algebraic system

The purpose of the numerical simulation is to show the effect of the algebraic nonlinearity, which on the one hand generates transients whose duration depends on the magnitude of the initial conditions, and on the other hand configures the nonlinear coupling with the outflow. The first simulation shows the effect of the initial conditions on the duration of the discharge time, and the second simulation shows the effect of the change of the opening in both valves.

The simulation is performed following the algorithm shown in the previous section, the numerical integration is done with the Euler method with an integration step of 0.01 seconds, the tanks are considered equal with a cross-sectional area of 0.0168 m^2 , the outlet valve is considered fixed with a value of $\alpha_3 = 0.001 \text{ m}^3 / (\text{s} \cdot \text{Pa}^{0.5})$ the tolerance error for the numerical solution is $\epsilon = 0.001$

Case A.- Two simulations with fixed valves, with different initial conditions.

In these simulations the valves are configured with $u_1 = 12 \times 10^{-5} \text{ m}^3 / (\text{s} \cdot \text{Pa}^{0.5})$ y $u_2 = 2 \times 10^{-5} \text{ m}^3 / (\text{s} \cdot \text{Pa}^{0.5})$ and a first simulation is carried out with initial conditions of the level of

liquid in $x_1(0) = 0.25 \text{ m}$ for the first tank and in $x_2(0) = 0.3 \text{ m}$ for the second tank. The second simulation has $x_1(0) = 0.35 \text{ m}$ and $x_2(0) = 0.5 \text{ m}$. The profiles of the liquid level trajectories are shown in Figure 4.

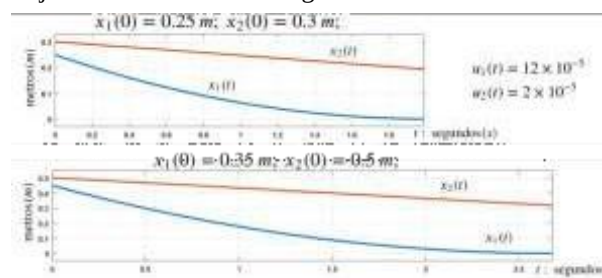


Figure 4. The higher the liquid level, the longer the discharge time.

Case B.- Time-variant valves

In this simulation case we have the following initial conditions $x_1(0) = 0.35 \text{ m}$ y $x_2(0) = 0.5 \text{ m}$ but the valves are configured as functions in time as follows:

$$u_1(t) = \begin{cases} 12 \times 10^{-5} \text{ m}^3/(s \cdot Pa^{0.5}), & 0 \leq t < 1.5 \\ 4 \times 10^{-5} \text{ m}^3/(s \cdot Pa^{0.5}), & t \geq 1.5 \end{cases} \quad \text{Eq (32)}$$

$$u_2(t) = \begin{cases} 2 \times 10^{-5} \text{ m}^3/(s \cdot Pa^{0.5}), & 0 \leq t < 0.5 \\ 10 \times 10^{-5} \text{ m}^3/(s \cdot Pa^{0.5}), & t \geq 0.5 \end{cases} \quad \text{Eq (33)}$$

Figure 5, has two plots, the first one shows the profiles over time of the valve 1 configuration expressed in Eq.

(32) and that of valve 2 expressed in Eq. (33). The other graph in Figure 5 shows the liquid level profiles of each tank.

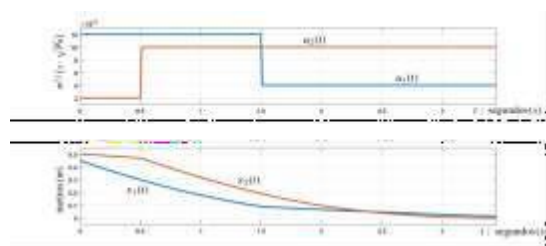


Figure 5. Effect of time-varying valves on tank levels.

Note that each tank provides the inlet pressures in the pipeline (see Fig. 3) and in addition to these pressures, the configuration of each valve defines the effect on the outflow, this effect over time is shown in Figure 6.

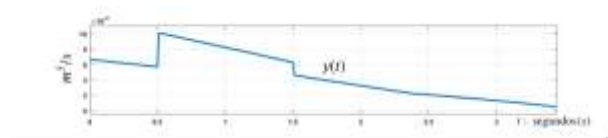


Figure 6. Effect of time-variant valves on outflow

DISCUSSION AND ANALYSIS OF RESULTS

The results of this article are dedicated to show an alternative of analysis for the confluence of liquids in pipes, particularly a pair of tanks is shown as a source of hydraulic potential, but with the necessary adaptations or properties, flow or pressure supplies of other characteristics can be approached. This same tenor is taken into in the model of the valves, since it is natural to think that there may be other properties or models in the algebraic part of the proposed model, and that may add algebraic solutions that could be solved with the same algorithm.

In contrast to the linear and invariant systems, it can be contrasted in Figure 4 that the trajectories have a duration sensitive to the initial conditions, since in the first simulation the first tank is emptied in 1.99 seconds, while in the other simulation the same tank is emptied in 2.68 seconds.

Figure 5 shows two plots, corresponding to the time-varying configuration profiles

In this simulation case, the second tank is emptied before the first one in a simulation time of 3.45 seconds.

In turn, Figure 6 shows the outflow profile through valve 3, which shows the abrupt increase in flow due to the increase in the opening of the second valve at 0.5 seconds, and subsequently there is an abrupt decrease in flow due to the partial occlusion of the first valve after 1.5 seconds. This profile obeys to the algebraic relationship that the outflow has with

valve configurations and with the liquid levels of each tank, which is expressed in the equation of y in the differential algebraic hydraulic system of Eq. (31).

CONCLUSIONS

The main parts proposed in this article are, on the one hand, the differential algebraic model of a hydraulic system, which considers as the algebraic part the solution of the conservative equation in a pipe in confluence with three nonlinear valves with their corresponding pressure differentials. And on the other hand, the proposal of the adaptation of a numerical solution algorithm by the domain bisection method, but formulated in such a way that it takes advantage of the properties of the algebraic expressions derived from the conservative property.

Future work that is naturally envisioned may be the consideration of a pair of valves in single-lever configuration to replace those that in this article can be considered as binando. Another interesting perspective is to add control loops for flow regulation, or even temperature regulation considering the tanks as sources of flow at different temperatures, making the analogy to the single lever valves for domestic use.

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