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RESONANCE PHENOMENON USING 3D PRINTING AND COMPUTATIONAL MECHANICS

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Abstract -- This manuscript presents a learning strategy for both classroom and laboratory settings. It summarizes the study of the dynamic response of cantilever beams with the aim of simulating vibrating rod tachometers. Using 3D printing, the prototype presented is a good option for studying the phenomenon of resonance. It is demonstrated that cantilever bars of the same length react in harmony when only one of them is excited by a single particular force.

Keywords: 3D printing, computational mechanics, resonance, mechanical vibrations, cantilever beam.

Abstract -- This manuscript demonstrates a learning strategy both in the classroom and in a laboratory environment. This is summarized in studying the dynamic response of cantilever beams with the aim of simulating vibrating bar tachometers. Using three-dimensional printing, the presented prototype is a good choice to study the resonance phenomenon. It is shown that cantilever bars with the same length react in harmony when only one of them is excited to a single particular force.

Key words – 3D printing, Cantilever beam, Computational mechanics, Mechanical vibrations, Resonance.

INTRODUCTION

The tuning fork is routinely used as an acoustic resonator. It provides a convenient means of producing a sound of a certain pitch, and therefore presents calibration opportunities, not only in musical instruments. The concept of resonance is exploited in a variety of contexts in scientific applications , including AFM (Atomic Force Microscopy) [1,2], MRI (Magnetic Resonance Imaging) [3], electrical circuits [4], lasers and optics [5], and nanotechnology [6]. As a fundamental physical phenomenon, it plays a key role in teaching vibrations. Furthermore, the unique advantages of additive manufacturing have been shown to encourage quality and learning from failures among engineering professionals and students in order to achieve designs that are feasible to manufacture. Generally, the evaluation of the results and functionality of creative designs using additive manufacturing is performed using an analytical or computational numerical method [7,8].

In this short article, we describe the use of a 3D printer to facilitate an effective demonstration of resonance. The device is self-contained and does not need to involve any measurements, as the demonstration can be based on purely visual appreciation.

Many conventional demonstrations of resonance involve the use of a variable (non-trivial) device or frequency driving signal, but the system described in this paper involves only simple ideas, direct observation, and a 3D printer.

The flexural stiffness of any thin structural member depends on the type of material and geometric properties, but especially on the length of the member, since the natural frequency tends to scale with the inverse of the square of the length [9]. For example, halving the length of a cantilever will typically quadruple the natural frequency. A continuous elastic system has an innate number of frequencies and mode shapes, but here we will focus on the lowest frequency in the bending vibration of prismatic cantilevers, i.e., members in which the cross-sectional properties are constant along the entire length.

Therefore, since all dimensions of various cantilevers can be printed with a high degree of precision and using the same material, it is

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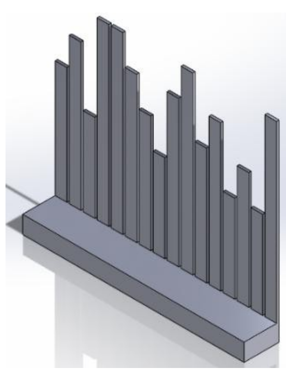
relatively easy to isolate a single parameter of change—the length. A detailed analysis based on the partial differential equation of motion can be explored, and such systems are also susceptible to approximation analyses such as Rayleigh-Ritz; such analysis is pending for further research.

3D printing has provided a useful means of exploring the stiffness and natural frequencies of aircraft structures [10,11], and given the ubiquity of 3D printers, it is not surprising that 3D-printed components can play a useful role in the field of physics and engineering education [12,13]. There may be deficiencies with some of the mechanical properties of ABS, a thermoplastic used by many 3D printers, for example, with viscoelasticity and accuracy in Young's modulus [14], but these are minimal for a single printed part.

DEVELOPMENT

A harmonic resonator consisting of flexible cantilevers, manufactured from ABS material, is designed and constructed (see Fig. 1). The educational prototype consists of four groups of bars of different lengths: 105 mm, 87.5 mm, 70 mm, and 52.5 mm. All have a rectangular cross-section of 1.51×7 mm. The mechanical properties of ABS are: Density=1060 kg/m^3, Young's modulus = 2275 MPa, Poisson's ratio = 0.35, bulk modulus = 2.52 GPa, shear modulus = 842.59 MPa, ultimate tensile strength = 40 MPa, and compression yield strength = 65 MPa.

Figure 1. CAD design of a flexible cantilever harmonic resonator (a) and 3D ABS print of the model (b).



(a)

(b)

Source: *Own creation.*

Analytically, the natural frequency of a flexible cantilever is determined by the equation [15]:

$$f_n = \frac{K_n}{2\pi}\sqrt{\frac{EIg}{wl^4}}, \qquad \text{Eq. (1)}$$

where E is Young's modulus, g is the force of gravity, l is the length of the bar, K_n is a factor that refers to the vibration mode (see Table 1), and w is the load per unit length, including the weight of the bar, and is defined by:

$$w = hb\delta g, \qquad \text{Ec. (2)}$$

where h is the height (1.51 mm), b *is* the width of the cross section (7 mm), and δ *is* the mass density of the cantilever bar material. The variable I in (1) is the second moment of inertia of the cross section, which for a rectangular section is determined by:

$$I = \frac{bh^3}{12}. \qquad \text{Eq. (3)}$$

Table 1. *Factors for the first 5 vibration modes of a cantilever beam.*

Vibration Mode	Kn
1	3.52
2	22
3	61.7
4	121
5	200

Source: *Own creation.*

For each bar of different length, the formula (1) is applied to determine the natural frequency in each vibration mode. The results obtained are shown in Table 2.

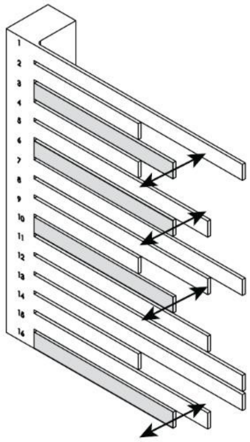
Table 2. *Results of the modal analysis for the first 5 vibration modes of the cantilever beam.*

Length (mm)	*Natural frequency per vibration mode (Hz)*				
	1	2	3	4	5
105	32.4	203	569	1120	1840
87.5	46.7	292	819	1610	2650
70	73	456	1280	2510	4150
52.5	131	821	2300	4510	7460

Source: *Own creation.*

Experimentally, if one of the bars of one of the 4 groups of length are displaced a certain transverse position in such a way as to generate a vibratory movement (see Fig. 2), the other three bars of the same length will begin to vibrate in unison. The same effect occurs if any of the other cantilevers are excited.

Figure 2. *Transverse displacement that generates vibratory motion in the cantilevered bars.*



Source: *Own creation*.

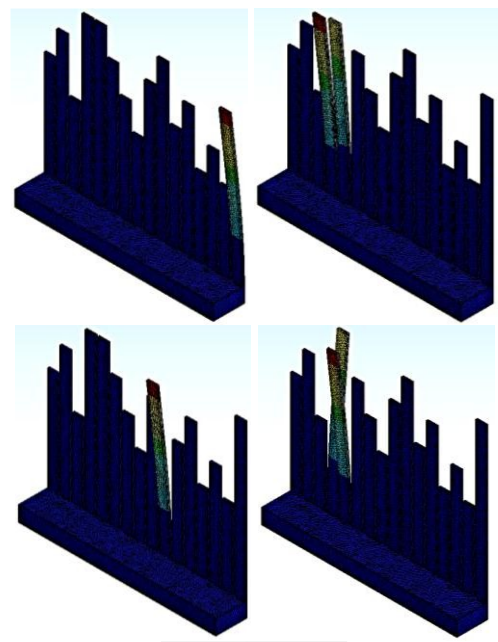
Therefore, the initially excited cantilevered bar causes the other bars of the same length to vibrate, thus generating natural frequencies that produce harmonic resonance. This is because the energy generated by the vibratory movement is structurally transferred through the printed base, especially due to the low damping coefficient. It can be observed that the mass or stiffness of the base is not a critical factor, as it is sufficient to simply hold the prototype in your hand and excite one of the bars to show the harmonic resonance effect. Furthermore, it is evident that longer cantilevers can produce larger amplitudes and lower resonance frequencies (see results in Table 2). Equation (1) reveals an interesting effect: the natural frequency of a cantilever is independent of its width. This is because the stiffness (specifically, the second moment of area, I) scales linearly with b, but so does the mass per unit length, so these cancel each other out.

DISCUSSION AND ANALYSIS OF RESULTS

A) Modal Analysis

The objective of modal analysis in structural mechanics is to determine the natural frequencies and modes of vibration of an object or structure during free vibration. It is common to use the Finite Element Method (FEM) to perform the analysis because, as in other calculations using FEM, the object being analyzed can have arbitrary shapes. The physical interpretation of the eigenvalues and eigenvectors, which come from solving the system, represent the corresponding frequencies and modes of vibration. Some of the results that can be obtained with this type of analysis are the deformation and high stress points in the structure due to each of the modes of vibration. Fig. 3 shows the free vibration modal analysis of the harmonic resonator prototype using FEM for the 105 mm long cantilever bar, in which the deformation of the four bars at a natural frequency can be observed. The software reported an average of 32 Hz, which corresponds to the first vibration mode. This value coincides with that calculated in Table 2. It is important to note that the software displays this average value as four different modes, but the numerical error between them is minimal and, therefore, only the deformations of all bars of that length are simulated. A similar explanation applies to the 87.5 mm and 70 mm long cantilevers, whose average natural frequencies for the first mode using computational mechanics are
46.01 Hz and 71.628 Hz, respectively. This again coincides with the results reported in Table 2. Figures 4 and 5 show the deformations for each of these groups of bars when they are in resonance.

Figure 3. *Deformation of the 105 mm cantilevers in a first vibration mode with an average frequency of 32 Hz.*

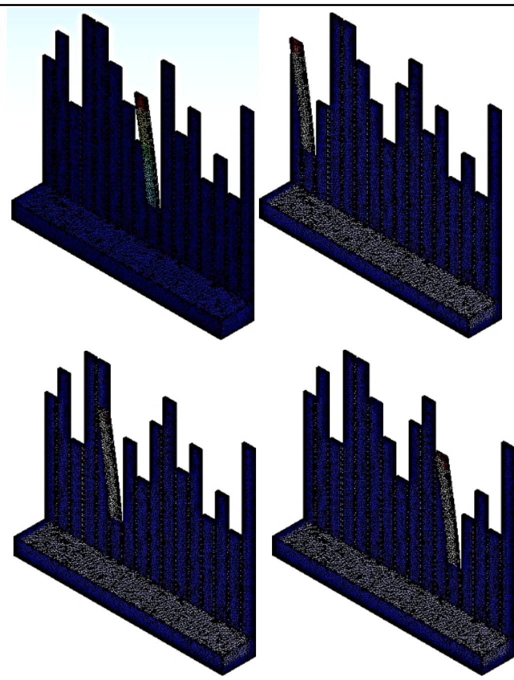


Tabular Data

	Mode	Frequency [Hz]
1	1.	31.983
2	2.	32.014
3	3.	32.017
4	4.	32.019
5	5.	45.997
6	6.	46.011
7	7.	46.016
8	8.	46.029
9	9.	71.574
10	10.	71.682

Source: *Own creation*.

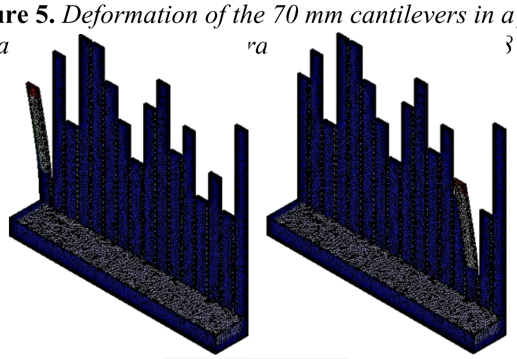
Figure 4. *Deformation of the 87.5 mm cantilevers in a first vibration mode with an average frequency of 46.01 Hz.*



	Mode	Frequency [Hz]
1	1.	31.983
2	2.	32.014
3	3.	32.017
4	4.	32.019
5	5.	45.997
6	6.	46.011
7	7.	46.016
8	8.	46.029
9	9.	71.574
10	10.	71.682

Source: *Own creation*.

Figure 5. *Deformation of the 70 mm cantilevers in a first vibra ... ra ... 3 Hz.*



	Mode	Frequency [Hz]
1	1.	31.983
2	2.	32.014
3	3.	32.017
4	4.	32.019
5	5.	45.997
6	6.	46.011
7	7.	46.016
8	8.	46.029
9	9.	71.574
10	10.	71.682

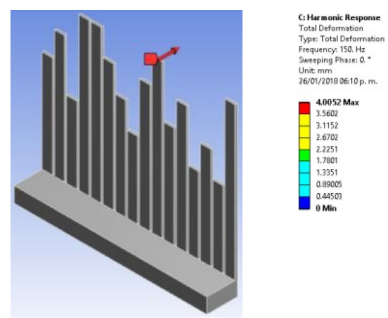
Source: *Own creation*.

Although Table 2 shows the values for the first five vibration modes of each cantilever, on occasion, the only modes desired are the corresponding to the lowest frequencies because they may be the predominant modes in the vibration of the object (which are those obtained for this prototype using FEM). Note: Due to the capacity of the department's computing resources, it was not possible to computationally model the 52.5 mm long cantilever bar.

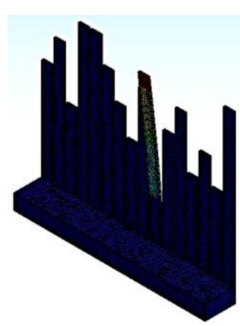
B) Harmonic analysis

Harmonic analysis is used to find the response frequency in a structure when it is subjected to an oscillating load in a certain vibration mode. This type of analysis is very useful because there are structures that have harmonic frequencies that are often not considered and can become an important failure factor. Among the results that can be obtained with this type of analysis is the amount of deformation suffered by a structure at a given frequency. Another important result is a graph showing the deformation of the structure within a defined range of frequencies. In view of the above, a harmonic analysis is simulated using computational mechanics for the harmonic resonator prototype. A load of 1N is applied to a 105 mm cantilever (see Fig. 6a) in a frequency range of 0 to 150 Hz, and a maximum deformation of 4 mm is observed (see Fig. 6b).

Figure 6. *Load application on one of the cantilevers (a) and deformation results from harmonic analysis (b).*



(a)



(b)

Source: *Own creation*.

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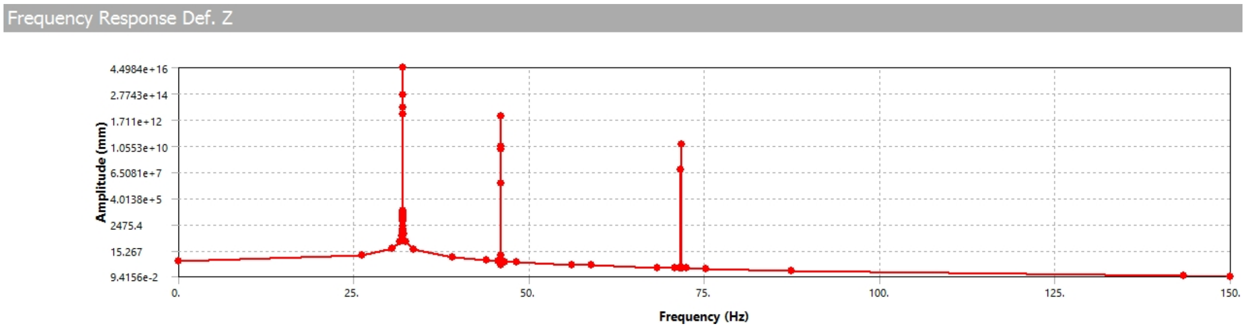
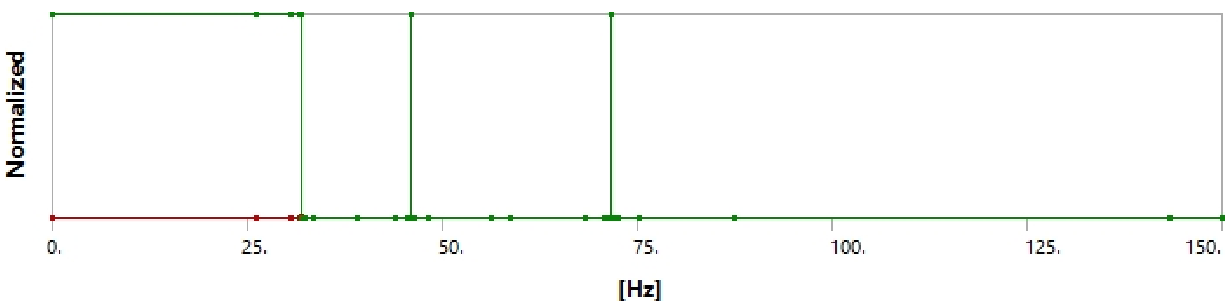
Figure 7. *Amplitude vs. frequency graph in the direction of load application.*





Source: *Own creation*.

Finally, Fig. 7 shows an amplitude *vs*. frequency graph in the direction of load application. This graph shows the critical deformation ranges of the harmonic resonator, which occur at the frequencies determined in the modal analysis (32 Hz, 46.01 Hz, and 71.628 Hz). Therefore, these are the resonance frequencies.

CONCLUSIONS

Conventional introductory material on mechanical vibrations commonly begins with the mass-spring system, despite the relative difficulty of changing parameters, such as adding another mass. However, for systems with distributed mass and stiffness, simple cantilevers have been used to easily demonstrate the natural frequency of vibration through observation. This paper shows a simple extension, using a 3D printer, to develop an effective demonstration of resonance to characterize one of the most important parameters in structural dynamics and vibration. Future lines of work in educational research can focus on:

- Designing a prototype from a significant set of cantilever overhangs, following a periodic pattern of slightly different lengths that functions as a basic mechanical tachometer device. To indirectly determine the operating frequency (*rpm*) of a systemic component of a machine (e.g., an engine) from the critical vibration of one of the cantilevers. Provided that the dominant excitation frequency is within the range of natural frequencies, its values can be determined analytically and numerically as established in this work and validated experimentally by acquiring data using a laser tachometer at the tip of the cantilever involved.
- Study the spectrum of temporal dependence *versus* the displacement of the tip of the bar until the movement decays due to material damping and correlate it with its respective frequency spectrum defined from an FFT (Fast Fourier Transform).

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Contribution Role Table

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Methodology	Javier Flores Méndez / Gustavo M. Minquiz
Project management	Adriana Anel Téllez Méndez
Resources	Adriana Anel Téllez Méndez
Supervision	Gonzalo Espinosa Cuatlapantzi
Validation	Javier Flores Méndez / Gustavo M. Minquiz
Writing (Original draft)	Gonzalo Espinosa Cuatlapantzi / Marino Viveros Mora
Editing (Revision a nd editing)	Javier Flores Méndez / Gustavo M. Minquiz