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Abstract - The electrocardiogram (ECG) plays a fundamental role in the diagnosis of cardiac diseases, being one of the main causes of mortality worldwide. In the last decades, several techniques ECG signal processing have been developed, highlighting noise removal as a crucial factor to improve feature extraction. However, achieving even higher accuracy remains a persistent challenge. In this study, we present an innovative approach using an unbiased finite impulse response (UFIR) filter. Under noisy conditions and evaluating the root mean square error (RMSE) and signal-to-noise ratio (SNR), our proposed method shows remarkable performance compared to the weighted Savitzky-Golay (SG) filter. This work contributes to the continued advancement in ECG signal processing, providing the potential for more accurate and reliable detection of cardiac diseases.
Keywords: ECG, Signal estimation, Savitzky-Golay Fitlro, Smoothing filter, RMSE, UFIR, UFIR

Abstract -- Electrocardiogram (ECG) is of paramount importance in the diagnosis of heart disease and because it persists, it is the leading cause of death worldwide. Various techniques have emerged in recent decades to process ECG signals, and noise removal has played a prominent role in improving feature extraction. However, achieving greater accuracy remains an enduring challenge. This study presents an innovative approach that applies a weighted and unbiased finite impulse response (UFIR) filter. Under the same noise conditions and in terms of mean square error (RMSE) and signal-to-noise ratio (SNR), our proposed method shows decent performance compared to the weighted Savitzky-Golay (SG) filter. This research contributes to the progressive evolution of ECG signal processing, offering the potential for more accurate and reliable detection of heart disease. ***Key words -*** ECG, Signal Estimation, Fitlro Savitzky- Golay, Softener Filter, RMSE, UFIR

INTRODUCTION

An electrocardiogram (ECG) is a recording that reflects the electrical activity of the heart, being a crucial diagnostic tool for the detection of cardiac pathologies, [1]. The P-QRS-T waves present in the ECG signal provide valuable information, with the QRS complex being especially relevant in the identification of cardiac arrhythmias, [2]. Early detection of arrhythmias is vital to prevent and predict heart attacks, thus highlighting the importance of the ECG as a fundamental tool to save lives, [3]. However, the accuracy of arrhythmia detection can be affected by noise and artifacts, leading to misdiagnosis, [4]. To counteract this problem, a preprocessing step is essential. Preprocessing, a widely employed and essential procedure in ECG signal analysis, aims to reduce noise and improve signal quality, ensuring accurate extraction and diagnosis of cardiac features, [5]. Therefore, continuous monitoring of ECG signals over prolonged periods is required, employing analog and digital electronic devices for ECG data acquisition and processing.

Currently, several techniques have been proposed to remove noise and artifacts in ECG signals, especially smoothing methods. Recent studies have demonstrated the efficacy of

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based digital wavelet transform and conventional low-pass and band-pass filters in noise reduction in digitized biomedical signals using embedded systems. In one of these studies, a unique methodology combining wavelet-based transform with wireless IoT devices to monitor cardiac behavior was introduced, [6]. Another approach employed adaptive Fourier-Bessel domain wavelet transform automatic detection of anxiety stages using the signal from a single-channel portable ECG sensor, [7]. In addition, different types of filters have been evaluated, such as low-pass Butterworth type filters with fourth and eighth order, compared to filters such as Chebyshev type, [8]. In other studies, the use of artificial intelligence (AI) techniques based on deep learning has been proposed to classify and reduce the noise associated with ECG signals [9],[10],[11]. Also, the application of generative adversarial neural networks (GANs) has been suggested for blind restoration of ECG signals, other applications associated with classification and pattern detection,[12],[13],[14],[15],[16]. However, these techniques are just beginning to emerge and their computational cost is really high. Therefore, it is necessary to optimize the parameters used in AI for the reduction of associated noise in ECG signals.

It is important to note that, due to the difficulties inherent in the a the techniques mentioned for ECG signal processing, a viable alternative lies in the implementation of polynomial filters. These are easy to apply and provide improved estimation of biomedical signals. The Savitzky-Golay (SG) smoothing filter is commonly used ECG analysis, although it has limitations in its parameters, such as the need for an odd length for the horizon parameter [17],[18]. Despite these limitations, it remains a standard method for removing noise in ECG signals. Another polynomial model based filter is the UFIR (*Unbiased Finite Impulse Response*) p-shift filter which has been widely used to remove noise in ECG, highlighting its adaptability for slow and fast behaviors. It has been used to estimate temporal morphological features of the ECG, [19],[20],[21],[22]. Therefore, the UFIR filter presents important properties that can result in an improvement in the estimation of ECG signals. The aim of this work is to develop a comparative analysis between Savitzky- Golay (SG) and UFIR polynomial filters with weights to their gain factors. Therefore, this research focuses on the development of a robust iterative weighted UFIR filter designed to enhance ECG signals and compare it with the weighted SG filter obtaining interesting preliminary results. This work is organized in the following sections: Section II describes the different methods used in this study. In Section III, we elaborate on the key findings, evaluated through RMSE and SNR metrics, with a additional focus on the practical implementation of real ECG signals. Finally, the last section describes the conclusions of the work.

DEVELOPMENT

Methodology

The methodology implemented for the development of this work is shown in Figure 1. Initially, a synthetic ECG signal is considered, to which a random noise (AWGN*, additive White Gaussian noise*) is added at a given decibel level. This ECG signal with noise is the input of the UFIR and SG filters with weights to be evaluated means of RMSE and SNR analysis. The evaluation process is performed several times in order to find the best filter by analyzing the filter response and stability. In the following sections, the concepts and results associated with the mathematical model of the UFIR filter and its algorithm are explained.

Figure 1. Methodology for finding the best filter.

State-space model of the ECG signal

The discrete-time model for representing ECG signals is given in [5], where the signal is represented on an interval horizon $[m,n]$ of length N, where $m= n-N+ 1$. The polynomial grade used in the representation is determined at state state, providing an accurate representation of ECG signal within the specified time frame. The ECG signal is time invariant and deterministic. The ECG signal measurement is assumed to be corrupted by zero mean noise with an unknown standard deviation and a non-necessarily Gaussian distribution. Under such conditions, an ECG signal can be represented as follows:

$$\mathbf{x}_k=\mathbf{A}\mathbf{x}_{k+1} \qquad \text{Eq. (1)}$$

$$y_k= \mathbf{C}\mathbf{x}_{k+1}+ v_n \qquad \text{Eq. (2)}$$

where x_k is the process vector of the ECG signaly_k is the observation of ECG signal measurement,v_k is the zero-mean measurement noise with unknown distribution, is the observation matrix defined as **C**= [10 . . . . 0] and the defined matrix**A** is the system matrix represented as follows:

$$\mathbf{A}=\begin{bmatrix} 1 & \tau & \frac{(\tau)^{2}}{2} & \cdots & \frac{(\tau)^{K-1}}{(K-1)!} \\ 0 & 1 & \tau & \cdots & \frac{(\tau)^{(K-2)}}{(K-2)!} \\ 0 & 0 & 1 & \cdots & \frac{(\tau)}{(K-3)!} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \quad \text{Eq. (3)}$$

Knowing the above variables, over a horizon [m,n] of a series of ECG points, the UFIR fitro can be represented as:

$$\boldsymbol{x_k}= (\boldsymbol{W}^T_{m,n}\, \boldsymbol{W}_{m,n})^{-1}\boldsymbol{Y}_{m,n} \quad \text{Eq. (4)}$$

Where$Y_{m,n}$ is the extended measurement vector of the ECG signal and the matrix W_mn is known as the augmented matrix, both matrices can be represented as follows, respectively:

$$Y(m,n)= [y^T_m\; y^T_{m+1}\; \ldots\; \ldots y^T_n\,]^T \quad \text{Eq. (5)}$$

$$\boldsymbol{W}_{m,n}= \begin{bmatrix} C(A^{n-m})^{-1} \\ \vdots \\ CA^{-1} \\ C \end{bmatrix} \quad \text{Eq. (6)}$$

Iterative UFIR filter with weights

In the following, we present an algorithm (see Algorithm 1) based on the iterative filter with weights whereY_{ms} is the ECG signal with noise (measurement of the ECG signal), where

the variable N=$N_{opt}$is the horizon or window of points represented by the following equation:

$$q_{opt} = \frac{N_{opt}-1}{2} - \frac{1}{2}\sqrt{\frac{N_{opt}^2+1}{5}} \quad \text{Eq. (7)}$$

Algorithm 1: Iterative UFIR filter algorithm Data: Y,N = N_{opt} , A, C, W
Result:
1: Start
2: for $k=N-1,N$ **... do**
3: $m=k-N+1, s=k-N+K$;
4: $G_s= (W^T_{m,s}W_{m,s})Y_{m,s})$;
5: $\hat{x}_s=G_s\,(W^T_{m,s})(Y_{m,s})$;
6: $i=s+1:k$ **do**
7: $\tilde{x}_i=A_i\tilde{x}_{i-1}$;
8: $G_i= [C^T_iC_i+(A_iG_{i-1}A^T_i)^{-1}]^{-1}$;
9: $K_i=G_iC^T_i$
10: $\tilde{x}_i=\tilde{x}_i + K_i\,(y_i-H_i\tilde{x}_i)$
11: end for
12: $\hat{x}_n=\tilde{x}_n$
13: $x_{n-q}=A^{-q}_n x_n$
14: end for
15: Result$\hat{x}_k$

In addition, this iterative UFIR smoothing algorithm similar to the Kalman filter is presented in [23] recursively in two distinct phases: prediction and update. This algorithm recalibrates the generalized noise power gain (GNPG), and its adjustment depends on a gain derived from processing by

batches. Unlike the Kalman filter, the UFIR algorithm described in Algorithm 1 has a significant advantage in that it does not require any

prior knowledge about the measurement noise. This feature endows the UFIR algorithm with a higher level of robustness and reliability. In [24], [25], an improvement of the UFIR filter robustness with the weight γ, defined by

$$\gamma=\frac{1}{\lfloor N/2\rfloor}\sum_{i=k_0}^{K}\sqrt{\eta_i/\eta_{i-1}} \quad \text{Eq. (8)}$$

$k_0=k-\lfloor N/2 \rfloor+1$, the term $\lfloor N/2 \rfloor$ is the integer part of and$K$ the number of states. The root mean square deviation (RMS) of the estimate is calculated.

using the innovation residue as:

$$\eta_k=\sqrt{\frac{1}{K_m}(y_k - C_k\hat{x}_k)^T(y_k-y_k\hat{x}_k)} \quad \text{Eq. (9)}$$

WhereK_m is the dimension of the target motion, y_k is the ECG signal measurement,C_k is the state vector and$\hat{x}_k$ is the ECG signal estimate.

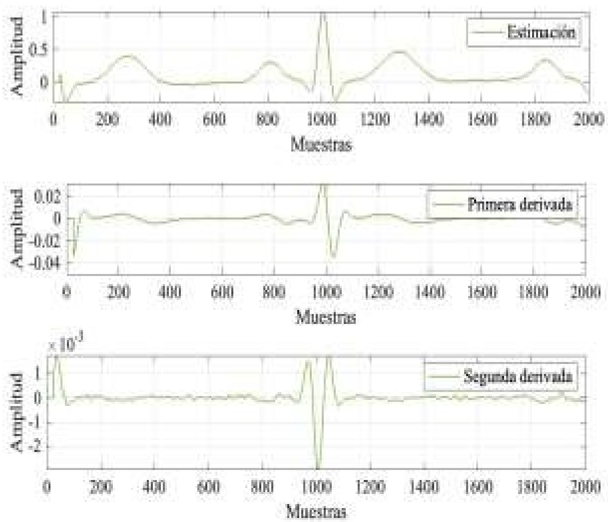

Figure 2. State estimation produced by the UFIR iterative UFIR filter.

Savitzky-Golay filter for ECG signals

The SG filter can be considered as a special case of the UFIR smoothing filter, as shown in []. The convolution-based smoothed estimate with a mid-horizon offset is given by $p=\frac{N-1}{2}$

$$\hat{s}_{k|k-\frac{N-1}{2}}=\sum_{n=-(N-1)/2}^{(N-1)/2}\varphi_n y_{k-n} \qquad \text{Eq. (10)}$$

Whereφ_n (-p) represents the convolution coefficients determined by the method of minimums.

linear squares (LS) to set up with commonly low-degree polynomial systems. The coefficients h^i_n can be extracted from the FIR function explained in []. It is important to mention that the SG filter has the following restrictions to ensure accurate calculations, the length of the horizon$N$ must always be an odd number. If an even number is used, the summation limits would include fractional values, which is not desirable.

While the fixed delay is generally set as, it is important to note that different applications may require alternative delay values. The optimum delay may not be

Implementation of UFIR and SG filters

Considering a 3X3 process matrix A represented as:

$$A=\begin{bmatrix}1 & 1 & 0.5\\ 0 & 1 & 1\\ 0 & 0 & 1\end{bmatrix} \qquad \text{Eq. (11)}$$

an observation vector C=[1 0 0], a horizon N_{opt}= 41 and aq_{opt} , calculated by equation (7), Algorithm 1, we can estimate the ECG signal, for three

states. The first state is the estimate of the ECG signal, the second state is the first derivative of the ECG signal and the third state is the second derivative of the ECG signal (See Figure 2).

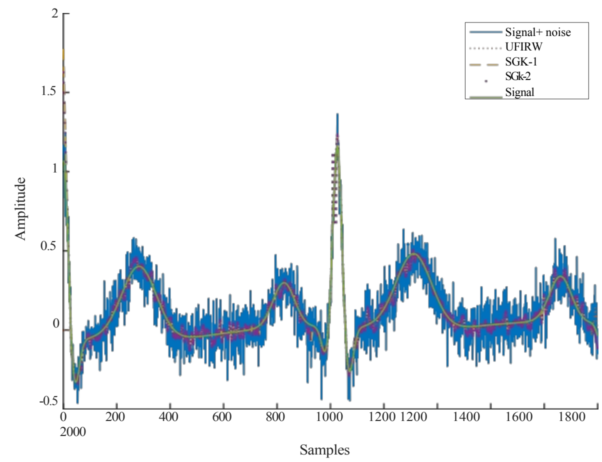

Figure 3. Estimates obtained by the filters studied.

For the SG filter, the following parameters were considered, N=41, Kaiser windows of weights with a value of 38 and 48, as shown in Figure 3.

visulizar the estimation of the filters studied considering a synthetic ECG signal.

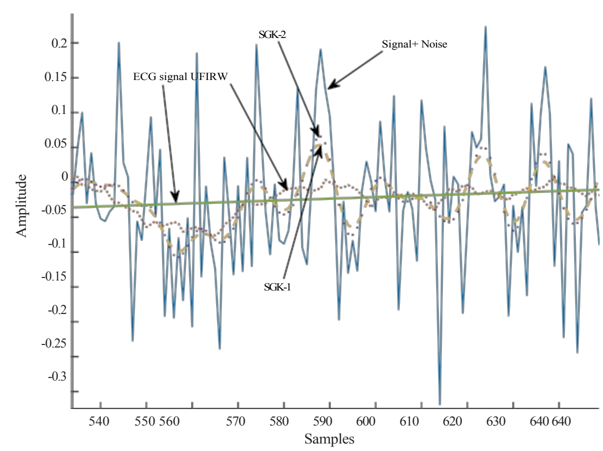

Figure 4. Estimation obtained for a specific part of the ECG signal by the filters studied.

This implementation was developed under the Matlab platform version 2023 with MAC M2 Processor. Figure 4 shows in detail the behavior of the estimations in each of the filters. The weighted UFIR filter with weights (UFIRW) and

Savitzky-Golay based filters with the Kaiser weighting windows mentioned above, SGK-1 and SGK-2 estimate a part of the ECG signal.

DISCUSSION AND ANALYSIS OF RESULTS

RMSE analysis

The estimation of the ECG signal by the studied filters is compared in terms of RMSE. The root mean square error is determined by equation 6.

$$RMSE=\sqrt{\frac{1}{L}\sum_{i=0}^{L}(\tilde{x_i}-y_i)^2} \qquad \text{Eq. (12)}$$

where $\tilde{x_i}$ is the ECG signal sample estimated by the filters, y_i is the ECG reference signal sample, and L is the size of the samples. Under 1000 iterations and a random noise of -20dB in figure 5 can be visualize the RMSE performance of each of the filters, where the UFIR-based filter obtained a superior performance with an average value of 0.01, compared to the performance of the SG-based filters obtaining minimum and maximum values between 0.015 and 0.035 respectively.

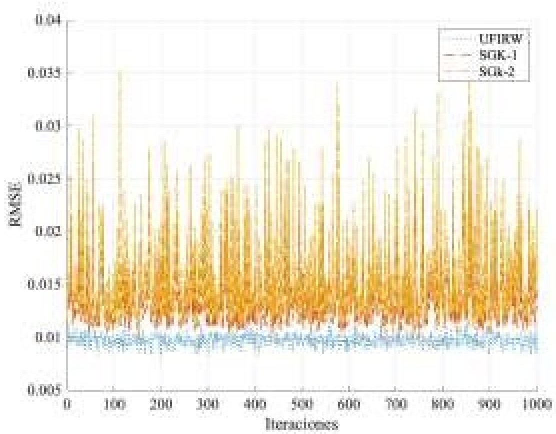

Figure 5. RMSE of the different filters used.

In addition, a further study is made on statistics of the performance of the filters by means of their RMSE. In Figure 6, the concentrations and dispersions of the RMSE in each filter can be visualized by means box plots. The 25%, 50% and 75% quartiles presented in Table 1 reflect the RMSE concentration of the analyzed filters. According to this study, it is observed that SG-based filters generate more outliers than those based on UFIR.

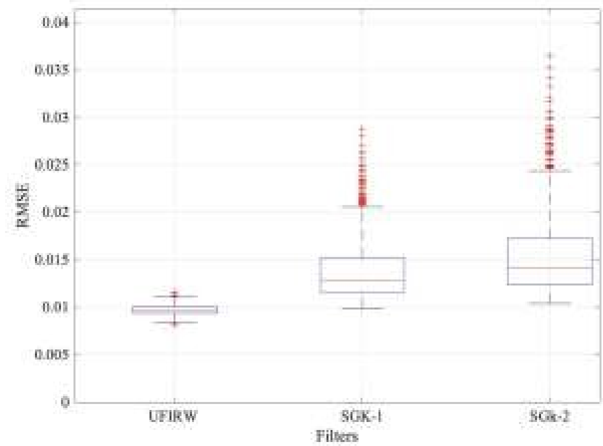

Figure 6. Visualization of the different RMSE concentrations for each filter used.

Table 1. of associated with the concentration of the filters studied.

Filter/Statistics	UFIRW	SG-1	SG-2
Quartile 25% Quartile 25% Quartile 25% Quartile	0.0093	0.011	0.012
Quartile 50% Quartile 50% Quartile 50% Quartile	0.0097	0.028	0.014
Quartile 75%	0.0100	0.015	0.017
Outliers	12	58	56

SNR Analysis

The signal to noise ratio (SNR) analysis was performed under different noise levels in the range of -50 to 50 dB with white Gaussian noise (AWGN) (see Figure 7). the noise level varies the UFIR based filter is superior to the Savitzky-Golay based filters.

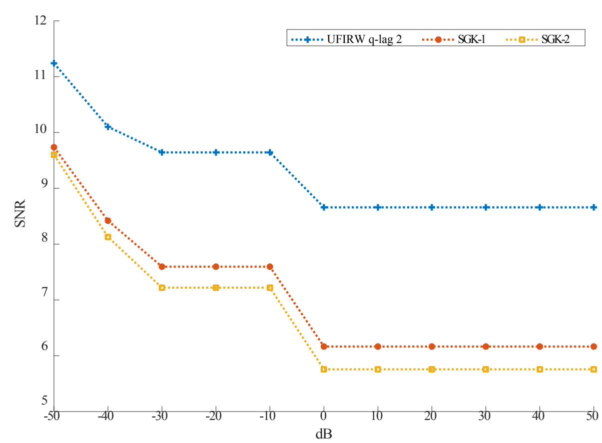

Figure 7. SNR analysis of the used filters

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Estimation of real ECG signals

A variety of real ECG signals obtained from the MIT-BIH Arrhythmia database were analyzed [26],[27]. By employing the weighted UFIR filter, accurate estimates were achieved for a variety of real signals presenting different cardiac pathologies.

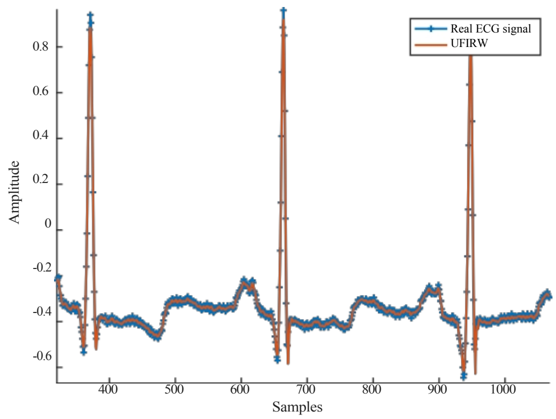

Figure 8. Estimation of the UFIR filter in real signals with normal sinus rhythm.

As can be seen in Figure 8, the filter estimates an ECG signal with normal sinus rhythm. Here, we can determine that no alterations occur in the original signal associated with the morphological characteristics of the ECG signal.

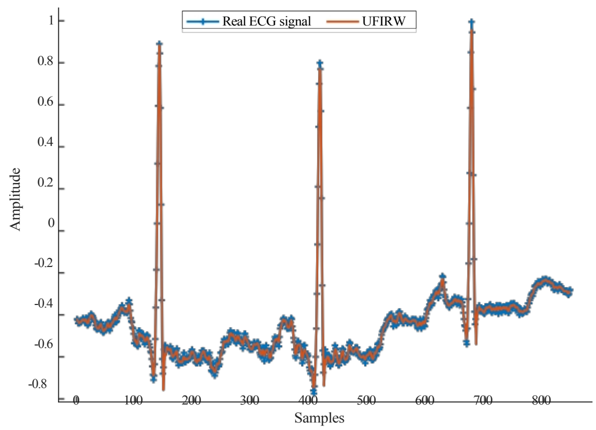

Figure 9. Estimation of the UFIR filter on real ECG signals with alteration of their baseline.

When observing Figure 9, the filter estimates an ECG signal with normal sinus rhythm with alterations in its baseline. We can visualize that there are no alterations in the original signal associated with the morphological characteristics. In addition, improvements can be made on the baseline correction by estimating the baseline by increasing the horizon of N points.

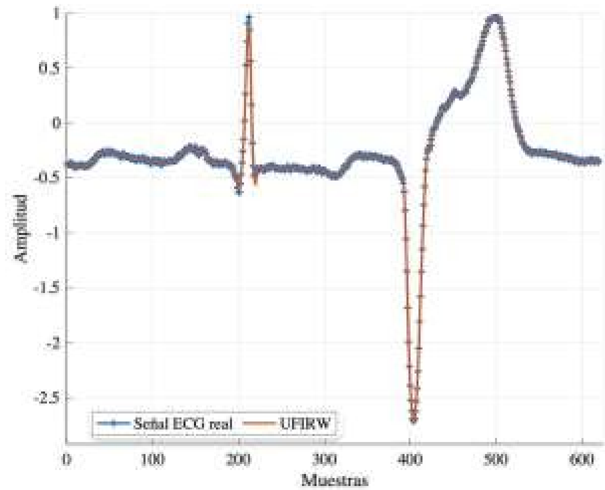

Figure 10. UFIR filter estimation in ECG signals of a ventricular premature complex.

Finally, Figure 10 shows the estimation of an ECG signal with a premature complex.
atrial, which highlights morphologic features that may determine pathology.

Discussion

The results disclosed above provide detailed insight into the impact of the unbiased finite impulse response filter with weights (UFIR) on ECG signal noise attenuation, contrasting with Savitzky-Golay based filters with weights (SG). This evaluation offers a unique opportunity to explore the significant implications and contributions of our findings in the current context of cardiac signal processing techniques. Existing literature has highlighted the need to effectively address noise in ECG signals due to their vulnerability to external variations and perturbations. However, the diversity of approaches and the continuous search for more efficient methods underline the urgency of a rigorous and comparative evaluation of available techniques. In this sense, our study set out to analyze the performance of the UFIR filter in comparison with the SG filter, focusing not only on noise attenuation, but also on its stability in the face of random variations. From the analysis performed, a deeper understanding of the practical and theoretical implications of our findings was reached.
Throughout this section, we will explore connections to previous research, highlight key contributions of our work, and possible practical applications of these results in the field of cardiac signal processing. In addition, we will address limitations identified during the study and point out directions for future research arising from our observations.

The findings of this study in applying the UFIR filter in reducing ECG signal noise are consistent with previous research that has highlighted the importance of addressing noise in cardiac signals. Research in [4], [5], [6],[21] , 22] and [23] demonstrated in their investigation the effectiveness of adaptive filters in noisy conditions, and our results support this notion by showing a significant improvement in noise attenuation compared to conventional Savitzky-Golay based filters with weights (SG).

In contrast, the results obtained with the SG filter align with previous studies that reported limited effectiveness in terms of noise attenuation in Analysis that reinforces these observations by demonstrating that, while the SG filter may be effective in certain contexts, the UFIR offers greater stability and more consistent noise reduction under conditions of random variation. These findings support the idea that careful selection of filtering methods is essential, and the versatility of the UFIR filter, especially in the presence of random noise, positions it as a promising option in ECG signal processing. Our research contributes to the existing literature by providing additional evidence on the relative efficacy of different filtering approaches, highlighting the need to consider the specific nature of noise when selecting electrocardiographic signal processing methods.

CONCLUSIONS

The weighted UFIR filter outperforms the Savitzky-Golay filter in terms of root mean square error (RMSE) and SNR analysis, emerging as a promising strategy for real electrocardiographic signal processing.

For future work, various pathologies related to ECG cardiac recording will be explored. These pathologies will be analyzed to extract features in the time and frequency domain, with the aim enabling their automatic detection using artificial intelligence techniques. It should be noted that the algorithms investigated in this study, together with Internet of Things (IoT)-based devices, can be applied to other biomedical signals, which could be fundamental to identify specific patterns of pathologies and to monitor vital signs in patients.

This research proposes two future lines of work: Digital Medical Signal Processing and Smart Medical Devices. The first line will focus on the development of advanced algorithms to filter biosignals, considering the variability caused by noise and artifacts. The second line will focus on the implementation of artificial intelligence algorithms in cyber-physical systems, allowing the identification of patterns useful for the diagnosis and prediction of pathologies.

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